Classification of Chain Rings

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a joint work with

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Abstract

An associative Artinian ring with an identity is a chain ring if its lattice of left (right) ideals forms a unique chain. Let R be a chain ring with invariants p, n, r, k, k', m. In this article, we first prove that for every chain ring R, there exists a certain finite commutative chain subring which characterizes it. Using this fact, we classify chain rings with invariants p, n, r, k, k', m up to isomorphism by finite commutative chain rings ($k' = 1$). Thus the classification of chain rings is reduced to that of finite commutative chain rings.

Keywords

local ring, chain ring, Galois ring, p-adic field, isomorphism class.

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